Homework 5
STA4322/STA5328
Introduction to Statistics Theory
Spring 2020, MWF 3:00pm
Due date: Monday, March 9th, 2020

All work must be shown for complete credit. W.M.S. denotes the course textbook, *Mathematical Statistics with Applications*.

1. Let $U_1, \ldots, U_n$ be a random sample from the uniform distribution on $(0, 4\theta)$. Derive the method of moments estimator of θ.

2. Suppose that $X_1, \ldots, X_n$ are independent and identically distributed random variables having a Poisson distribution with mean θ.
   (a) Find the MLE for θ.
   (b) Prove that $\hat{\theta}$, the MLE, is consistent for θ.
   (c) What is the MLE for $P(X_i = 0) = e^{-\theta}$?

3. Let $Y_1, \ldots, Y_n$ be a random sample from a distribution with density
$$f_Y(y \mid \theta) = \begin{cases} \theta y^{-2} & \theta \le y < \infty \\ 0 & \text{otherwise} \end{cases},$$
   where $\theta > 0$.
   (a) What is a sufficient statistic for θ?
   (b) Find the MLE for θ.
   (c) Find the method of moments estimator of θ.

4. Let $Y_1, \ldots, Y_n$ be a random sample from a distribution with density
$$f_Y(y) = \begin{cases} \left(\frac{2}{\theta^2}\right)(\theta - y) & 0 \le y \le \theta \\ 0 & \text{otherwise} \end{cases}.$$
   (a) Use the method of moments to construct an estimator of θ.
   (b) Is this estimator a sufficient statistic for θ?

5. Let $W_1, \ldots, W_n$ be independent and identically distributed random variables having one of two densities. If $\theta = 0$, then
$$f_W(w \mid \theta) = \begin{cases} 1 & 0 < w < 1 \\ 0 & \text{otherwise} \end{cases}$$

and if $\theta = 1$, then

$$f_W(w \mid \theta) = \begin{cases} \frac{1}{2\sqrt{w}} & 0 < w < 1 \\ 0 & \text{otherwise} \end{cases} .$$

Compute the maximum likelihood estimator of θ.

6. Let $x_1, \ldots, x_n$ be fixed, known constants and let β be a fixed, unknown constant. Let $\epsilon_1, \ldots, \epsilon_n$ be independent and identically distributed random variables where $\epsilon_i \sim \mathrm{N}(0, \sigma^2)$, for $i = 1, \ldots, n$ with $\sigma^2 > 0$ unknown. Suppose we observe pairs $(y_1, x_1), \ldots, (y_n, x_n)$ where

$$y_i = \beta x_i + \epsilon_i, \quad (i = 1, \ldots, n).$$

   (a) Find a two-dimensional sufficient statistic for (β, σ^2).

   (b) Find the MLE for β, and verify that it is unbiased for β.

   (c) What is the distribution of the MLE for β?