Homework 3
STA 4321/5325, Fall 2019, MWF 8:30am
Professor: Aaron J. Molstad
Due date: Wednesday, September 18th, 2019

All work must be shown for complete credit. W.M.S. denotes the course textbook (*Mathematical Statistics with Applications*). Note that #5 is worth 2pts of extra credit (graded all or nothing): it is not required.

1. Suppose that when a machine is properly adjusted, 50 percent of the items produced by it are of high quality and the other 50 percent are of medium quality. Suppose, in addition, that the machine is improperly adjusted during 10 percent of the time and that, under these conditions, 25 percent of items produced by it are of high quality and 75 percent are of medium quality.

   Suppose that five items produced by the machine at a certain time are selected at random and inspected. If four of these items are of high quality and one item is of medium quality, what is the probability that the machine was adjusted properly at that time?

2. (W.M.S., 2.115) Five identical bowls are labeled $1, 2, 3, 4, 5$. Bowl i contains $5 - i$ black balls, with $i = 1, 2, 3, 4, 5$. All other balls are white (there are five balls total in each bowl). A bowl is randomly selected and two balls are randomly selected (without replacement) from the contents of the bowl. Using the laws/theorems/rules from lecture and defining relevant events:

   (a) What is the probability that both balls selected are white?

   (b) Given that both balls selected are white, what is the probability that bowl 3 was selected?

3. Suppose that a box contains six blue and four red balls. If five balls are selected at random, without replacement, determine the probability mass function of the number of red balls which will be obtained.

4. (Similar to W.M.S., 3.12) Let X be a random variable with $P(X = x)$ given in the accompanying table:

x	1	2	3	4
$P(X = x)$	.3	.4	.1	.2

   Find the following: (a) $\mathrm{E}(X)$, (b) $\mathrm{E}(1/X)$, (c) $\mathrm{Var}(X)$, and (d) $\mathrm{E}(X^2 - 1)$.

5. (Optional) Seven letters are distributed randomly into seven mailboxes. Let $X_i =$ the number of mailboxes containing exactly i letters. What is the probability distribution of X_3? (That is, find $P(X_3 = x)$ for every possible x.)