Homework 2 STA 4321/5325, Fall 2019, MWF 8:30am Professor: Aaron J. Molstad Due date: Wednesday, September 11th, 2019

All work must be shown for complete credit. W.M.S. denotes the course textbook (*Mathematical Statistics with Applications*).

1. Let A and B be two arbitrary events. We know

 $0 < P(A \cap B) < P(A) < P(B) < P(A \cup B) < 1.$

You hope that both A and B occur, i.e., $A \cap B$ occurs. Which of the following would you be happiest to observe: that A occurred, that B occurred, or that $A \cup B$ occurred? Why?

2. Suppose A and B are independent events and P(B) < 1. Prove that

$$P(A^c \mid B^c) = P(A^c).$$

- 3. A pair of events A and B cannot be mutually exclusive and independent. Prove that if P(A) > 0 and P(B) > 0, then:
 - (a) If A and B are mutually exclusive, they cannot be independent.
 - (b) If A and B are independent, they cannot be mutually exclusive.
- 4. A box contains (*A*) coins with heads on both sides, (*B*) coins with tails on both sides, and (*C*) fair coins, i.e., coins with a head on one side and tail on the other. Suppose that the probability of selecting a coin with a head on both sides is 0.4, the probability of selecting a coin with a tails on both sides is 0.3, and the probability of selecting a fair coin is 0.3. If one coin is selected from the box at random and tossed once, what is the probability a head will be obtained?